R E S T R I C T E D

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REMARKS ABOUT R. P. FRASER'S

REPORTS ON JET RESEARCH

March 1944

R E S T R I C T E D

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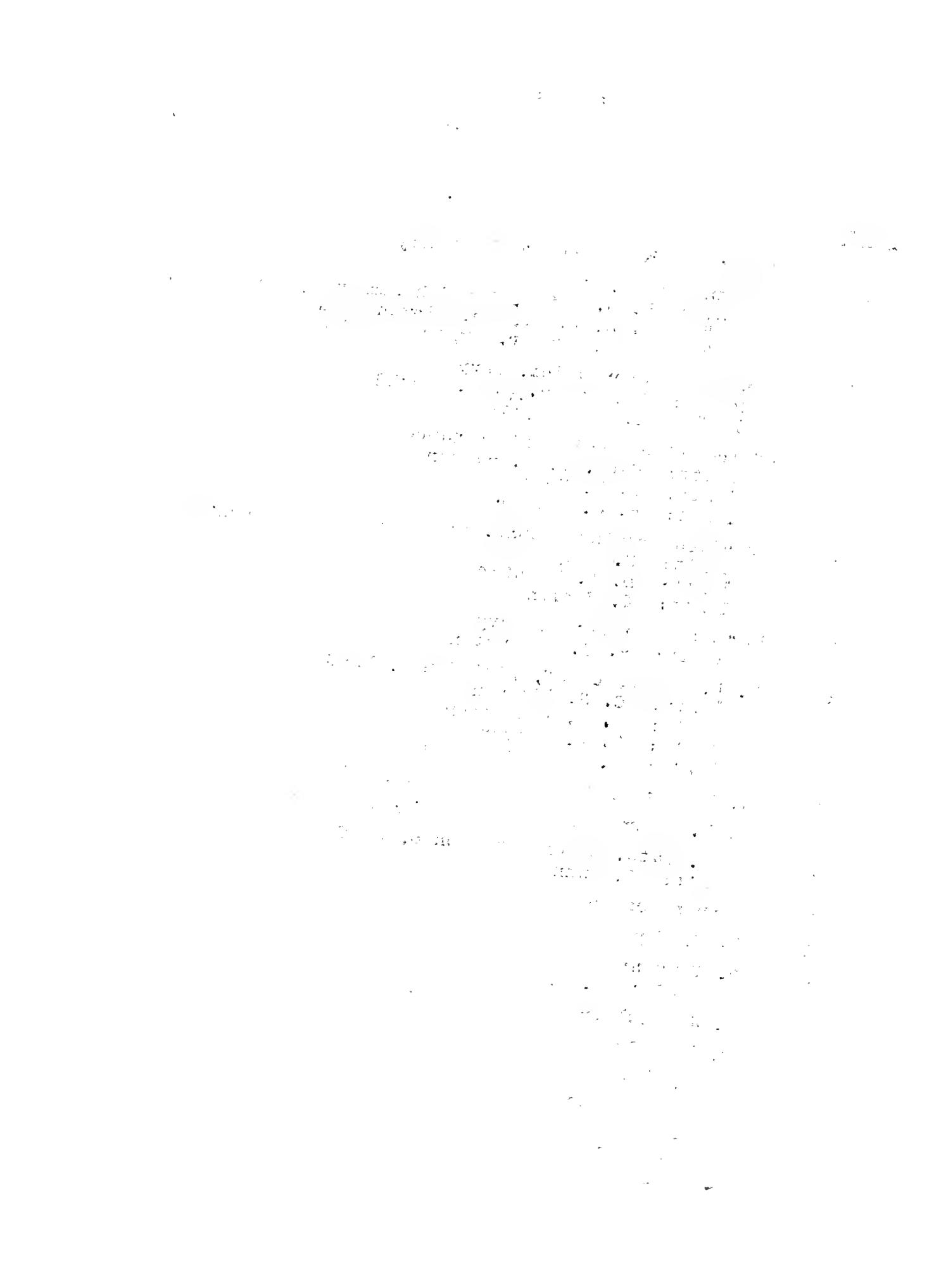
INTRODUCTION

The New York University Group of the Applied Mathematics Panel is grateful to the Liaison Office of the O.S.R.D. for providing a copy of R. P. Fraser's work, completed in the years 1940-41, which was needed in connection with a request from the Bureau of Aeronautics. The following analysis of Fraser's results (by K. O. Friedrichs) was carried out, in the light of more recent shock theory, in the hope of making these results useful for application to various problems of supersonic flow.

R. Courant

REMARKS ABOUT R. P. FRASER'S REPORTS ON JET RESEARCH

      The object of the present memorandum is to discuss
and to interpret experimental results presented by R. P.
Fraser[*] in a series of most remarkable shadow photographs
of the jet flowing out of exhaust nozzles and orifices at
high speed. These photographs, it seems, can be deciphered
more completely now than had been possible in the years
1940-41 since, in the meantime, a deeper theoretical under-
standing of possible patterns of shock and expansion waves
has been attained. On the other hand, a proper interpreta-
tion of experimental results such as those by Fraser might
be important for the investigation of phenomena inside ex-
haust nozzles of rocket motors and other devices.

      Under this point of view those results by Fraser
will be analyzed here which seem connected with matters
of some actuality.

      The first report (July 1940) concerns flows
through nozzles involving various kinds of changes such

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[*]Fraser, R. P.: Flow through Nozzles at Supersonic Speeds -
           Four interim reports on Jet Research, July 1940
           to June 1941.

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as a grid inscribed in the nozzle, a relatively long par-
allel section at the throat, a co-axial ledge at the throat,
nozzles whose divergent and convergent parts are out of line.
The result is that all these changes are not seriously detri-
mental to the jet. Also multiple nozzles are investigated;
they involve considerable losses. (Photos 78 to 110 belong
to this series). Photo 78, e.g., shows very clearly the
Mach lines in the jet resulting from slight disturbances
at the nozzle wall.

Of more specific interest for us is the second
report (Oct. 1940 and Jan. 1941). A set of nozzles with
half-angles of divergence from 5° to 30° are investigated.
All these nozzles have the same throat area and the same
exit area (in ratio 1 to 6). These areas correspond, on
the basis of the hydraulic theory, to a ratio 60 to 1 of
chamber to exhaust pressure: i.e., if the chamber pressure
is 60 atm then one expects the jet to have expanded to
just 1 atm at the exit. The contour line of the nozzle
consists of two sections. The convergent entry section
is an arc of a circle which just attains axial direction
at the throat; the divergent section of the contour is a
straight line; (i.e., the divergent section of the nozzle
is conical). This design was chosen for simplicity in
manufacturing and adjustment; it implies, of course, a
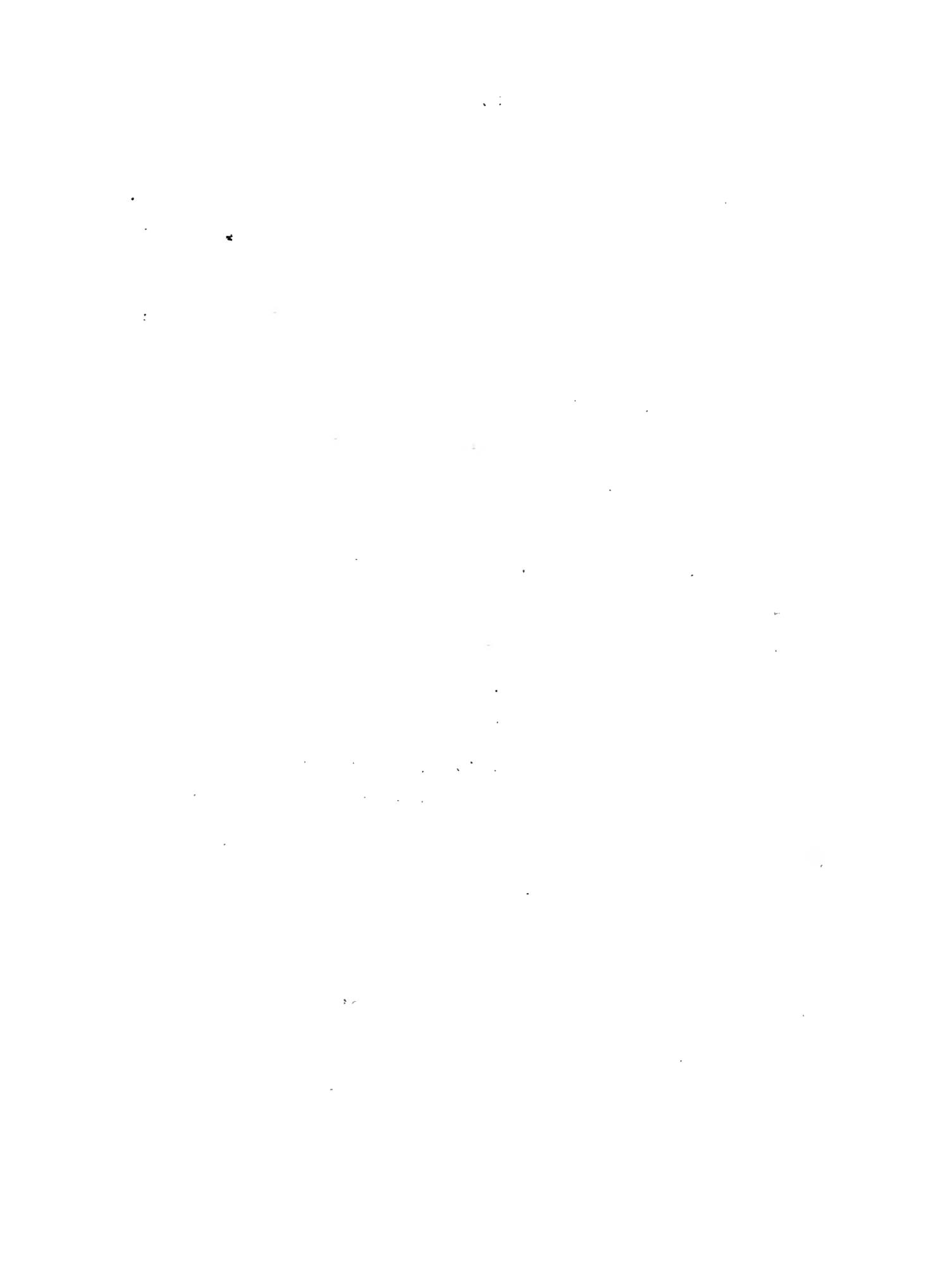
considerable change in direction at the throat (from 5° to 30°). Air under pressures ranging from 1200 to 200 lb/in^2 (i.e., 80 to 13 atm) was fed to the chamber and the resulting jet emerging from the nozzle mouth was photographed. The jet was apparently steady.

For the interpretation of the photos, the following considerations are essential. Suppose a conical surface (or any surface of revolution) encloses air of greater density than that of the air outside. (See Table 1) Then light rays crossing perpendicularly to the axis will be so deflected that the image of the cone on the photograph consists of an unexposed (white) strip toward the inside of the cone contour followed by an overexposed (dark) region.[*] This applies to shock fronts facing outward (i.e., across which the air enters the interior of the cone) and to contact surfaces (i.e., stream surfaces separating regions of different density). The same applies to rarefaction wave regions when the air stream crosses them from interior to exterior: since rarefaction waves spread and do not represent sharp discontinuities the effect is less pronounced.

If the density inside of the cone is less than that outside, the image consists of an overexposed (dark) strip at the outside and an underexposed (light) region

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[*]Cf. Hilton, Proc. Roy. Soc. A 169, 174 (1938)
H. Lewy; B.R.L., Aberdeen, Rep. No. 373 (143)
Ph. C. Keenan, Expl. Res. Rep. No. 11 (Re2c)
Navy Dept. Bureau of Ord. Wash. (Rest.)

toward the inside of the cone contour. This applies to shock fronts facing inward; i.e., across which the air leaves the interior of the cone, and to contact surfaces.

A common feature of all the photographs of jets coming out of nozzles is a sharp white strip beginning at the nozzle rim. This strip is due to the fact that the boundary of the jet is a contact surface and it indicates that the density in the jet is greater than that of the outside air. (Greater density of the jet would imply lower temperature since the pressure is the same at both sides of the jet boundary. Probably the chamber temperature was not excessively high; in that case, the jet temperature should be very low indeed). As is expected, the discontinuity at the jet boundary develops into a boundary layer which gradually disintegrates; vortices are formed and the flow becomes agitated. This is indicated rather definitely in the photographs.

When the chamber pressure is greater than 900 lb/in^2, for which the nozzle was designed, the pressure in the jet at the mouth will be greater than one atmosphere. Hence, a rarefaction wave across which the pressure is adjusted to the atmosphere is expected to start at the nozzle rim. Such a wave seems indicated as a white strip in photo 48 (Table 2) which shows the jet coming out of the 5° nozzle (No. 24) (Table 1) working with 1170 lb/in^2 chamber pressure.

The rarefaction wave disappears when the mouth pressure is about 1 atm; and when the mouth pressure has fallen below 1 atm, a shock front is needed for adjustment. This is the situation for the flow shown in photo 49 (Table 2) corresponding to 800 lb/in^2 chamber pressure. A weak conical shock front begins at the rim which is "reflected" as a second conical shock front. It is interesting to note that the white strip which indicates the reflected shock front shows up much clearer than the light region preceding the dark line which indicates the "incident" shock front. This is in agreement with the predictions from the optical considerations made before. Photo 50 (Table 2) corresponds to 485 lb/in^2 chamber pressure. The incident shock front beginning at the rim, the "rim shock", is stronger and more inclined. The incident and reflected shock cones are truncated; the reflection does not take place at a point but through a "Mach shock front", a disk perpendicular to the axis. The cylinder of subsonic flow past the Mach disk is seen in the figure. The reflected shock is in its turn reflected at the jet boundary through a rarefaction wave, which is faintly indicated in the photo. When the chamber pressure is 214 lb/in^2, the jet has detached somewhere inside the nozzle as is clearly seen in photo 51 (Table 2).

It is interesting to observe that the jet at-
tains its minimum cross-section where the reflected
shock front is reflected. The flow through this cross-
section behaves somewhat like the flow across a nozzle
throat or rather out of an orifice; the flow changes
from subsonic to supersonic speed.

The flow pictures are similar for the 10° nozzle.
Photos 43 and 44 (Table 3) for the flow out of a 15°
nozzle (No. 27) exhibit new phenomena. For 375 lb/in^2
chamber pressure, one should have about 1 atm at the
mouth and expect only a very faint shock or rarefaction
wave to begin at the rim. A faint wave of this type
is seen to begin near the rim; it develops into a shock-
wave which becomes stronger when it approaches the
point of reflection, and the reflected wave has consider-
able strength, a most peculiar phenomenon which we shall
observe more clearly later on.

Another peculiar phenomenon is seen: A diverg-
ing weak shock front emerges from the interior of the
nozzle. It is not difficult to explain its origin.
The sharp change in direction at the throat forces the
jet to expand suddenly there. This expansion will be
stopped by a shock front, the "throat shock". The
throat shock will be reflected inside the nozzle and

part of this reflected throat shock extends to the outside of the nozzle. A similar throat shock pattern should occur in the 5° and 10° nozzles; but either these shocks are too weak to be observed or they have faded out in these long nozzles.

When the nozzle is shorter (or the divergence angle larger) the throat shock pattern should extend further out of the nozzle, and both incident and reflected shock should be visible. Strangely enough, this is not, or at most only faintly, seen in the 20° nozzle (No. 33), (Photos 39, 40, 41, 42), but it will be seen in Photos 35 to 38 discussed below. (Table 4)

In Photo 35, depicting flow out of a 22.5° nozzle (No. 32) with high chamber pressure 1030 lb/in^2, one can spot a rarefaction wave beginning at the rim. This wave is eventually stopped by a shock front of gradually in-creasing strength, the "stopping" shock front; the re-flected "stopping" shock front is of considerable strength. In Photo 38 which corresponds to the low chamber pressure of 240 lb/in^2 one observes that the jet has already de-tached from the wall in the interior of the nozzle. This becomes clear when one determines the position of the rim by comparison with a flow under high chamber pressure, Photo 35 say. Also, the boundary layer has already started

8.

disintegrating and the rim shock comes clearly from the
interior of the nozzle.

The throat shock can be observed very clearly
in Photos 35 to 38. The reflected throat shock is seen
faintly in Photo 36. Photo 37 presents a very interest-
ing flow pattern. The chamber pressure (425 lb/in^2) and
hence also the mouth pressure are smaller than in the flow
considered just before. The smaller pressure apparently
forces the rim shock to curve inward **more** strongly and
the reflection to take place nearer to the mouth. The
throat shock is affected in such a way that it suffers
a Mach reflection. The reflected throat shock soon
intersects the Mach disk of the rim shock; for lower
pressures (Photo 38) the Mach disk of the throat shock
nearly merges with the Mach disk of the rim shock.

There is no doubt that the incident throat shock
is weak while the reflected throat shock is strong; this
follows from the fact that the angle of the incident
throat shock with the axis is small while that of the
reflected throat shock is large. This fact is of con-
siderable interest. It is known that a given incident
shock can be reflected, if at all, in two ways, through
a weak and a strong shock. For progressing shock waves,
the strong reflection has been **rejected**. It is inter-

esting to see that for the steady flow out of a nozzle, which is the result of adjustment to a stable position, strong reflection is possible. It is likely, too, that the reflection of the rim shock (or the stopping shock) is of the strong variety, at least as long as the reflection is of the Mach type.
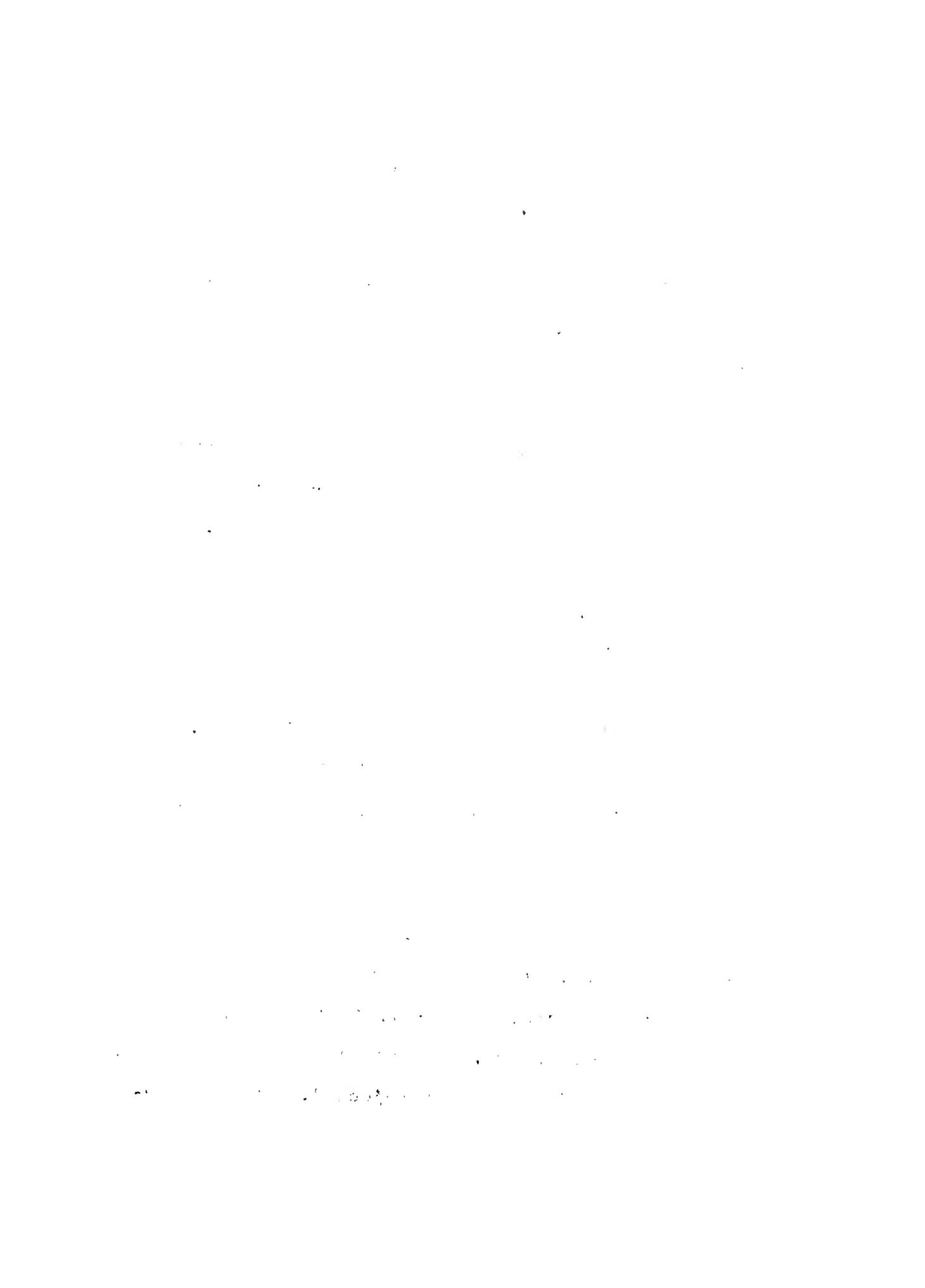
In Photos 31 to 34 depicting the flow out of the 30° nozzle (No. 31) one can just see the throat shock coming out of the nozzle (Photo 31) and its Mach disk which has nearly merged with the Mach disk of the stopping shock. In Photo 34 the Mach disk of the rim shock appears in a strange manner. It would probably be possible to explain this figure by unraveling the paths of light rays passing the neighborhood of this disk. Jet detachment in the nozzle is indicated for 370 lb/in^2 chamber pressure (Photo 34).

Our actual interest in Fraser's report derives mainly from his pressure measurements. The discussion of the flow pattern is to serve mainly for a proper interpretation of these measurements.

A hole was drilled through the nozzle wall near the mouth and the pressure of the air stream at the wall was measured. (We shall refer to the pressure at the point of tapping as "test point" pressure). It was found that the pressure ratio; i.e., the ratio of chamber pressure

to the test point pressure was nearly constant for
chamber pressure ranging from 1100 to 500 lb/in^2.
(The average value of this nearly constant ratio will
be called the average experimental pressure ratio). For
smaller chamber pressures, the test point pressure increased
rapidly. This was explained as indicating that jet detach-
ment occurs before reaching the test point. The mouth
pressure at which jet detachment first occurs is found
to lie between .5 atm and .65 atm rather independent of
the divergence. This is a remarkable result. One might
have expected that jet detachment would begin at lower
mouth pressures for long nozzles and at nearly 1 atm for
short and strongly diverging nozzles.

The measured values of the pressure ratios are
compared with those calculated on the basis of the hy-
draulic or one-dimensional theory for nozzle flow: (for
this calculation one need only know the chamber pressure,
the throat area and the cross-section area at the point
of tapping). At first, one should expect that the exper-
imental value of the test point pressure is greater than
the calculated value, at least for nozzles with consider-
able divergence, since divergence offers to the air
stream the opportunity of quick expansion. Certainly
this would be so for shockless and frictionless flow
through a smooth nozzle . Fraser's experimental results

however, show just the opposite, except for the 15° nozzle, as is seen from the table for the calculated pressure ratio R_{calc} and the average experimental pressure ratio R_{exp}. The table is copied below except that the values of R_{calc} and R_{exp}/R_{calc} were recalculated. (See also Table 5)

Nozzle	Half angle	R_{calc}	R_{exp}	R_{exp}/R_{calc}
31	30°	53.59	48.7	.91
43	25°	52.10	43.7	.84
32	22.5°	54.43	45.6	.84
33	20°	58.17	50.4	.87
27	15°	58.17	63.2	1.09
27a	15°	54.86	57.6	1.05
26	10°	58.87	57.9	.98
24	5°	57.53	56.05	.97

In interpreting these results one should not forget that the nozzles tested by Fraser involved a sudden change of direction at the throat. As a consequence of this disturbance, a shock front had developed, which was observed in some photos, corresponding to nozzles of 15° and wider opening. On crossing such a shock front the pressure increases suddenly and it may well be that this fact is responsible for the major portion of the 10 to 15 $^{\circ}/o$ increase in test point pressure observed in nozzles Nos. 31, 32, 33 according to the table above. In the some-

what longer 15° nozzle (No. 27), the flow past the shock
has apparently had enough chance to expand and thus to
decrease the pressure again; that the result amounts to
5 to $7^{\circ}/o$ more than the calculated pressure remains strange.
(A deviation of a few percent is of no significance since
the measurements cannot be expected to be more accurate than
2 or $3^{\circ}/o$.)   Whether or not the occurrence of shocks is
mainly responsible for the deviation from theoretical expecta-
tion is still the question. Fraser himself gives a differ-
ent explanation: "At 15° and greater divergences it may
be that the frictional loss in the convergent portion is
the more important part. This causes a consequent drop
in the pressure at the throat to below the theoretical
critical pressure and,therefore, a consequent drop in the
pressures along the divergence.   The rising mouth pressure
with increasing divergence must then be attributed to a
further departure from the adiabatic expansion in the
divergence due to general turbulence rather than wall fric-
tion."   Fraser's argument would also hold for nozzles with
a smooth throat. It would, therefore, be of interest to
have experimental evidence about the performance of smooth
nozzles with wide divergence.

          The preceding remarks refer to the pressure at
the wall. The flow distribution over the cross-section is
certainly influenced considerably by the shock fronts.

I. W. Maccoll, in a note added to Fraser's interim report
for February and March 1941, observes that better agreement
with the result of the hydraulic treatment could be expected
if, for comparison, the average pressure over a cross-section
is taken, instead of the wall pressure. He points out that
the photographs for the flow through the 15° nozzle show that
the pressure near the axis is greater than at the wall. This
is also clear from the fact that the air passes the observed
shock front from exterior to interior. The ratio R_{exp}/R_{calc}
would, therefore, be less than 1.07 if R_{exp} were based on an
average over a cross-section. For the 20° to 30° nozzles
the direction of the throat shock is such that the pressure
in the interior should be less than at the wall, and, conse-
quently, the ratio R_{exp}/R_{calc} should increase when based
on the average over a cross-section.

In any case, Fraser's pressure measurements are not
sufficient to derive conclusive information on the performance
of smooth nozzles with wide divergence.

A few indications may be given about the contents of
the third and fourth report. The experiments were repeated
with a new 15° nozzle (No. 27a) and the previous strange re-
sults were confirmed. A smooth 10° nozzle (No. 41) was tested
and the results were nearly identical with those for the
10° nozzle (No. 26) with change of direction at the throat

previously tested. It would have been interesting if smooth
nozzles with wider divergence had also been tested. A 10^0
nozzle (No. 42a) with a shorter entry section was tested
(Photo 55, Table 6); it is interesting that in this case
a reflected throat shock becomes visible. This shows that
the entry portion has an influence on the flow pattern even
in a long nozzle. A similar nozzle (No. 42) with a parallel
section at the rim was tested (Photo 56, Table 6). One ex-
pects that a shock front of some strength will develop at the
beginning of this parallel section and that the pressure at
the wall of this section rises considerably. Both expecta-
tions are clearly confirmed.

In the fourth report[*] a nozzle designed for the
rather low chamber pressure of 80 lb/in^2 = 5.44 atm was
tested (Photos 3 to 8, Table 7). For 30 lb/in^2 the shock
front has just come out of the nozzle, (Photo 3). For
80 lb/in^2 the emerging stream is rather parallel and only a
weak wave develops from the rim, (Photo 6). For greater
chamber pressures (100 and 160 lb/in^2) rarefaction waves
develop from the rim which are particularly visible. Again
these rarefaction waves are stopped by a shock front of
gradually increasing strength; this stopping shock is then
reflected by shocks of apparently considerable strength.

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[*] Another problem treated in this report may be briefly
mentioned: Obstacles were opposed to the jet, the
resulting flow pattern was photographed, and the some-
times very strange back reactions on the nozzle were
measured.

A gradually developing stopping shock with a rather strong reflected shock is particularly clearly seen in photo 1 (Table 6) depicting the flow out of an orifice; i.e., a nozzle which consists of only a diverging section, the exit being at the throat. If the pressure is increased, (Photo 24, Table 6), the stopping shock begins nearer the rim and becomes rather strong; the Mach disk nearly covers the whole jet cross-section.

The flow patterns in the jet as revealed by Fraser's Shadowgraphs differ decisively from the flow pattern described by Prandtl (1907). Prandtl's pattern for a jet which emerges from the nozzle with a pressure greater than atmosphere is easily described for two-dimensional flow emerging with constant velocity. Two rarefaction waves begin at the two edges through which the pressure adjusts to atmospheric pressure; these waves, after intersecting each other, are reflected at the opposite boundaries of the jet as contraction waves, which eventually converge to two points on the original boundaries; this flow pattern then repeats itself periodically. (See Table 8) It was apparently Prandtl's opinion that for jets emerging with non-constant velocity and for three-dimensional jets, the flow pattern would be similar. Stanton (1926) had concluded from various experiments that the actual flow pattern is quite different.

16.

Fraser's Shadowgraphs show the deviation of the actual pattern from Prandtl's pattern with particular clarity. V. Neumann, who in this connection had called the writer's attention some time ago to Fraser's photographs, had pointed out that the decisive new (and unexpected) element in the correct flow pattern is what was here called "stopping shock" of a rarefaction wave; that considerable over-expansion occurs in the front of the stopping shock (and the Mach shock); and that the stopping shock cuts through the Mach lines composing the rarefaction wave, beginning at the last of these lines in the direction of that line with strength zero. (See Table 8).

This strange phenomenon calls for an explanation. It is rather instructive for this purpose to compare Photos 48 and 31 which show the jet produced by essentially the same chamber pressure, 1140 and 1150 lb/in^2. Although the exhaust pressure is approximately the same in both cases, approximately 1.3 atm, the flow patterns are quite different. Stopping shock and reflected shock are much stronger in the jet with greater divergence; and it becomes clear that divergence has a strong influence on this phenomenon. It seems that the phenomenon can be explained as due solely to divergence, and the fact that atmospheric pressure must prevail at the jet boundary. It is evident that in a diverging flow the pressure will continue to decrease and very rapidly at that. The reason is that the stream will continue in
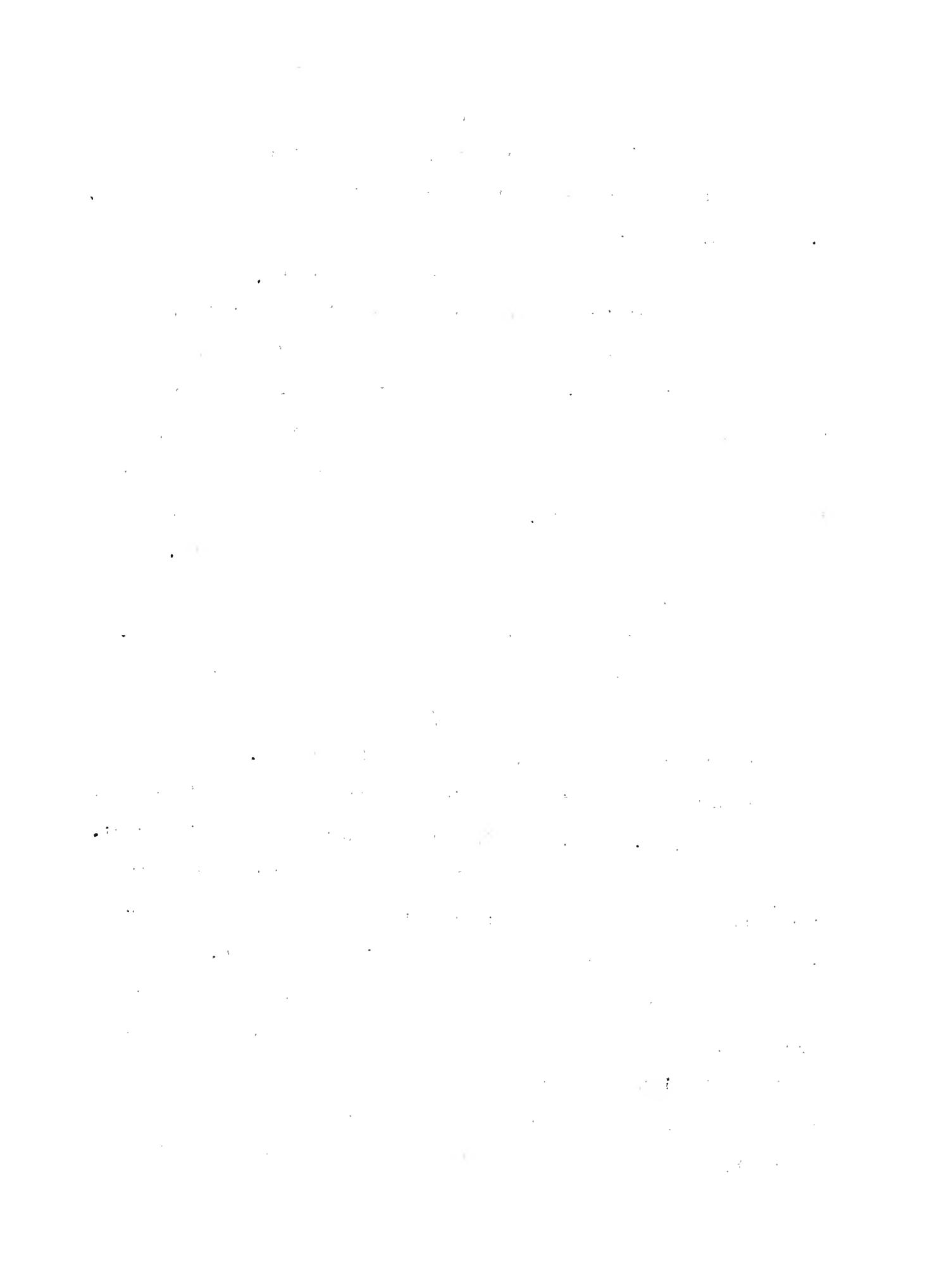
the same way that it would if the nozzle continued, until it reaches the first Mach line of the rarefaction wave. Across the rarefaction wave the pressure decreases further and it is thus clear that there will be a pressure gradient from the jet boundary toward the inside of the jet. This pressure gradient then will cause the jet to turn inward. Since the pressure is constant along the jet boundary, the Mach angle there is also constant. As a consequence, the Mach lines issuing from the jet boundary under the Mach angle turn inward. Although it is not excluded by the present argument that these Mach lines eventually diverge again, it appears that they do not, but rather intersect. It could be seen that the first point of intersection: i.e., the tip of the envelope of these Mach lines, lies on the external Mach line of the rarefaction pencil. Thus it follows that a shock front must begin somewhere past the rarefaction wave and cut into it. That the shock should begin at the rim is not indicated; various of Fraser's flow pictures could be so interpreted that it begins later. The arguments advanced could be corroborated and refined by considering the hodograph of the flow.

18.

In this connection, a paper by J. Hartmann and F. Lazarus should be mentioned (The Air-Jet with a Velocity Exceeding that of Sound, The Philosophical Magazine, XXXI, January, 1941), in which many photographs of jets from an orifice are shown and pressure measurements are reported. It was found that the pressure within the jet decreases strongly to a value far below atmospheric pressure until raised by a shock front. A feature of particular interest can be seen from the photographs: when the chamber pressure is small, a stopping shock begins near the opposite boundary of the jet. In that case the shock front consists only of a truncated cone, which corresponds to what had been called "reflected" shock front. As the chamber pressure is increased, the truncated cone becomes a full cone and, on further increase, the Mach disk and what had been called the "incident" shock front appear. It is clear that the "incident shock" is weak shortly after it has appeared, and on this basis it could be seen that the reflected shock is of the strong variety. These results seem to corroborate the arguments given above.

The latter phenomena did not occur in Fraser's shadowgraphs. There is no doubt, however, that on closer scrutiny they would reveal much additional information on possible jet patterns.

TABLE 1

NOZZLES OF VARIOUS
DIVERGENCES
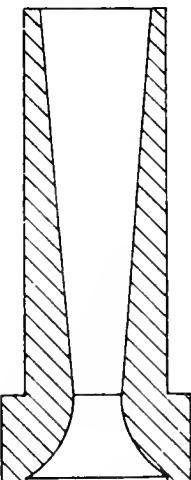
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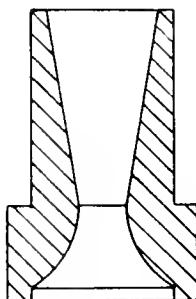
No. 26 - 10°
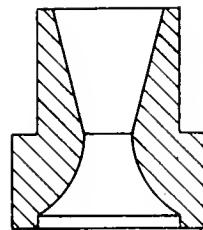
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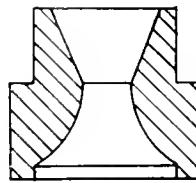
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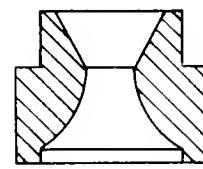
No. 31 - 30°

IMAGE OF CONE OF GAS

light
rays

Cone of gas

cone contour
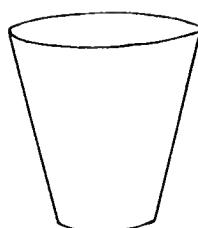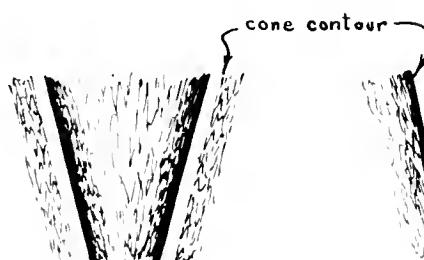
Image of cone when
interior is _more_ dense
than exterior
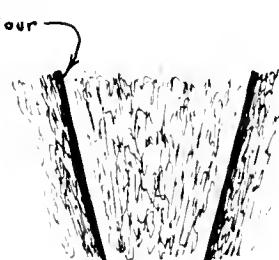
Image of cone when
interior is _less_ dense
than exterior

TABLE 2
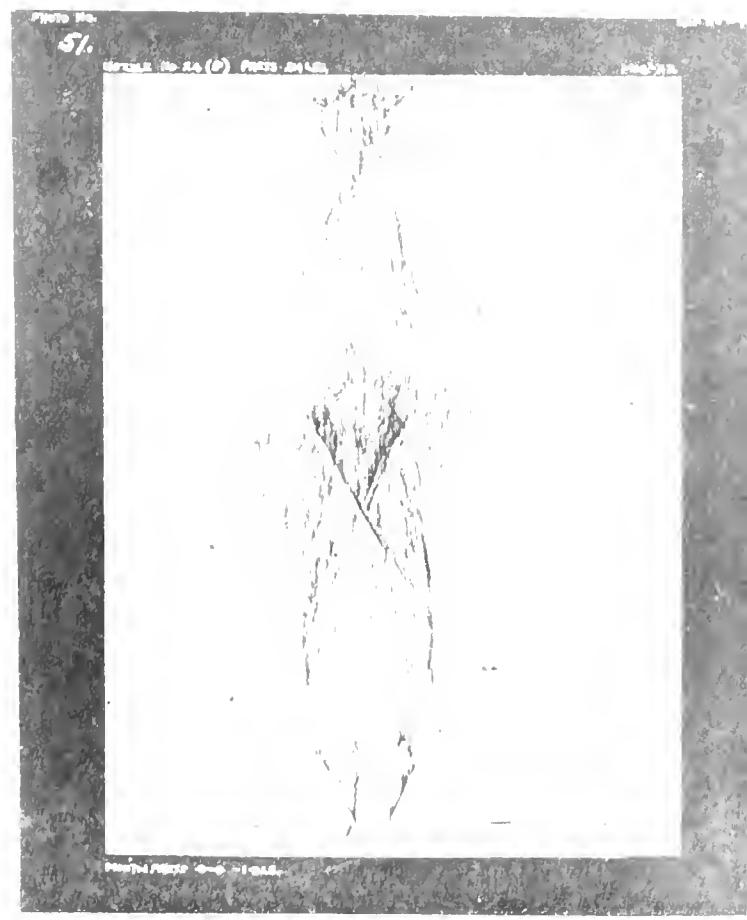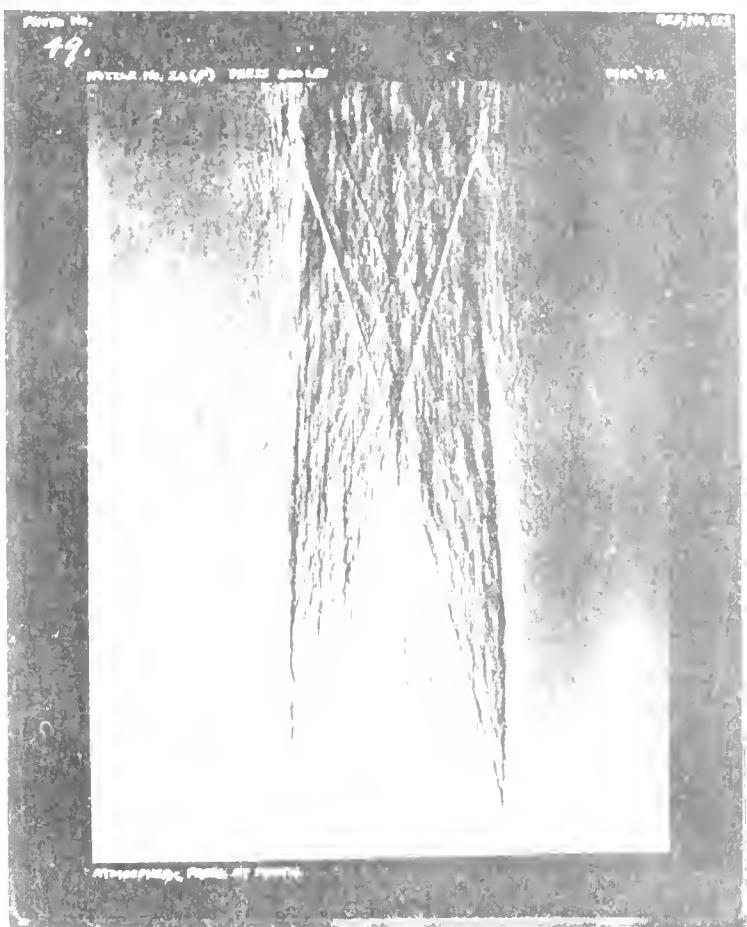
TABLE 3
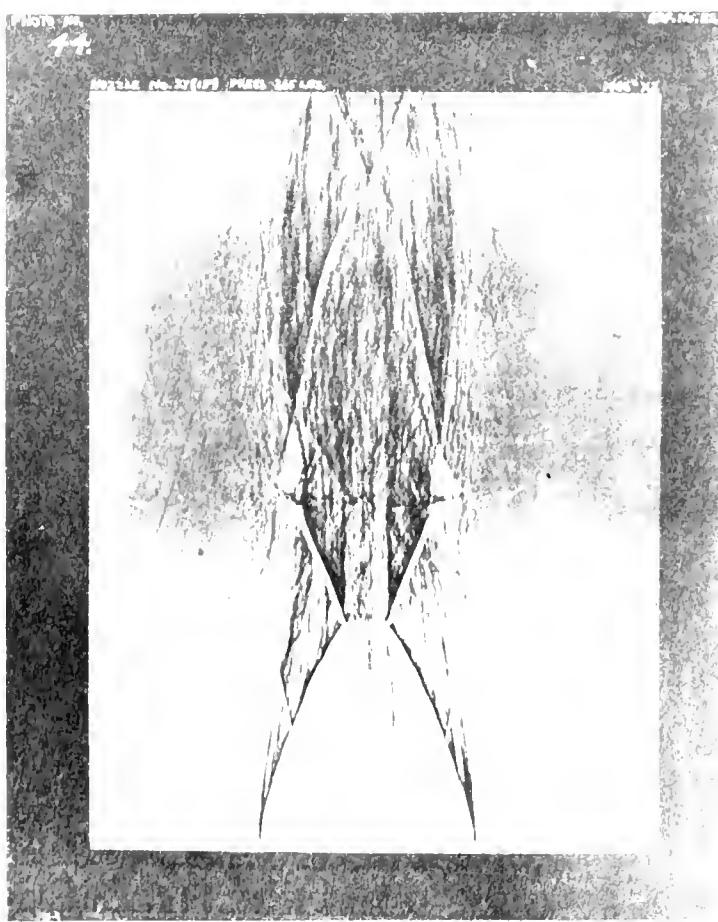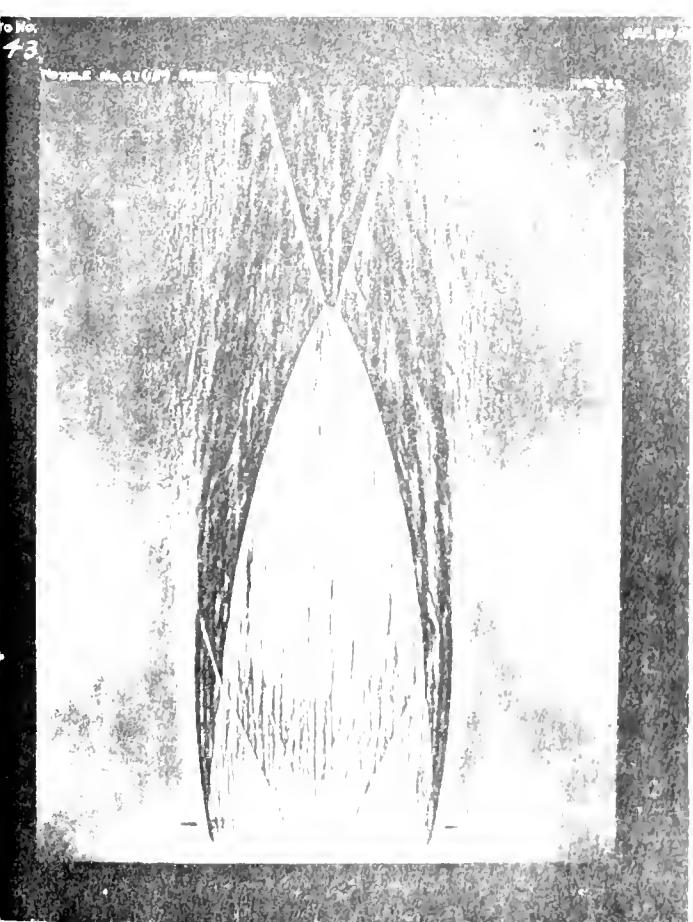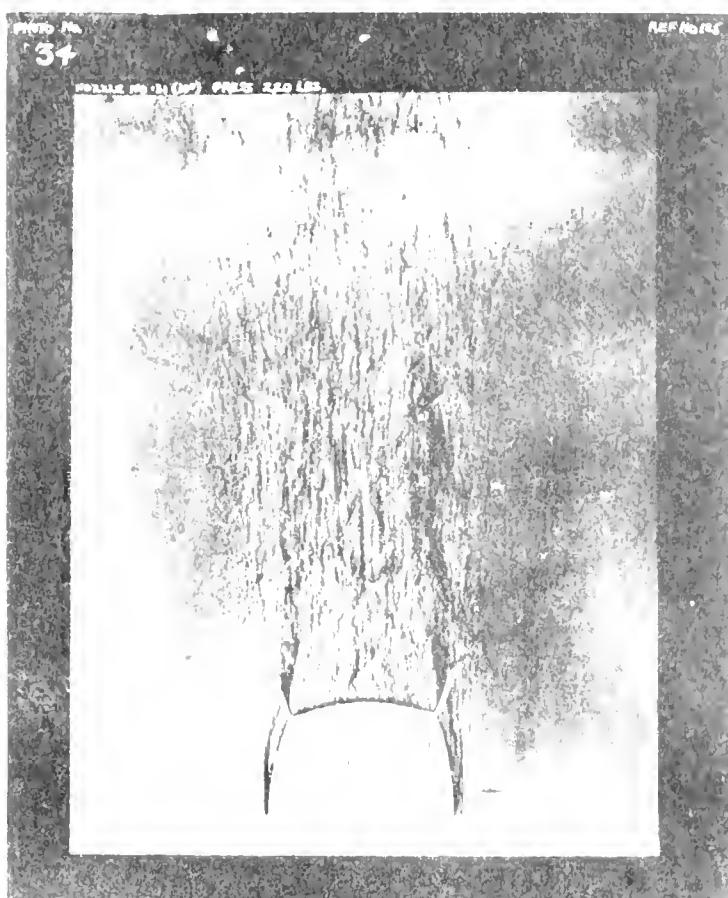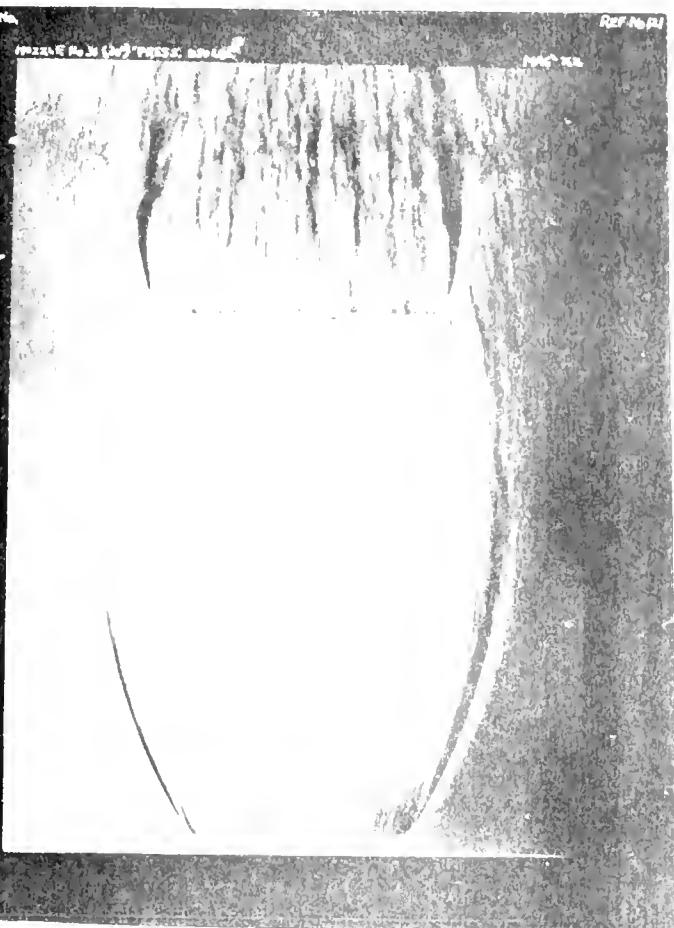
TABLE 4
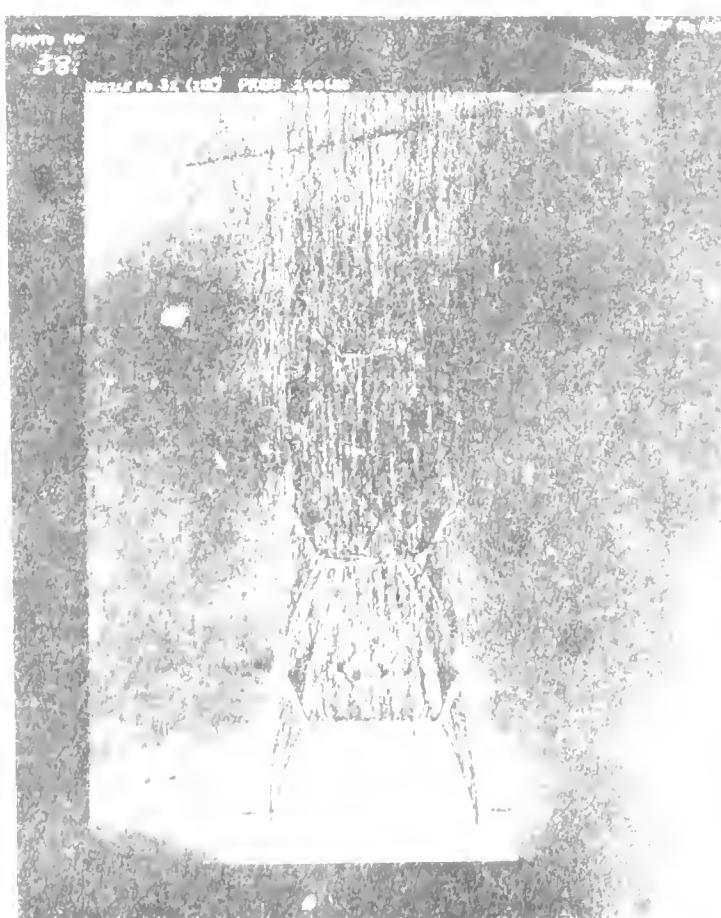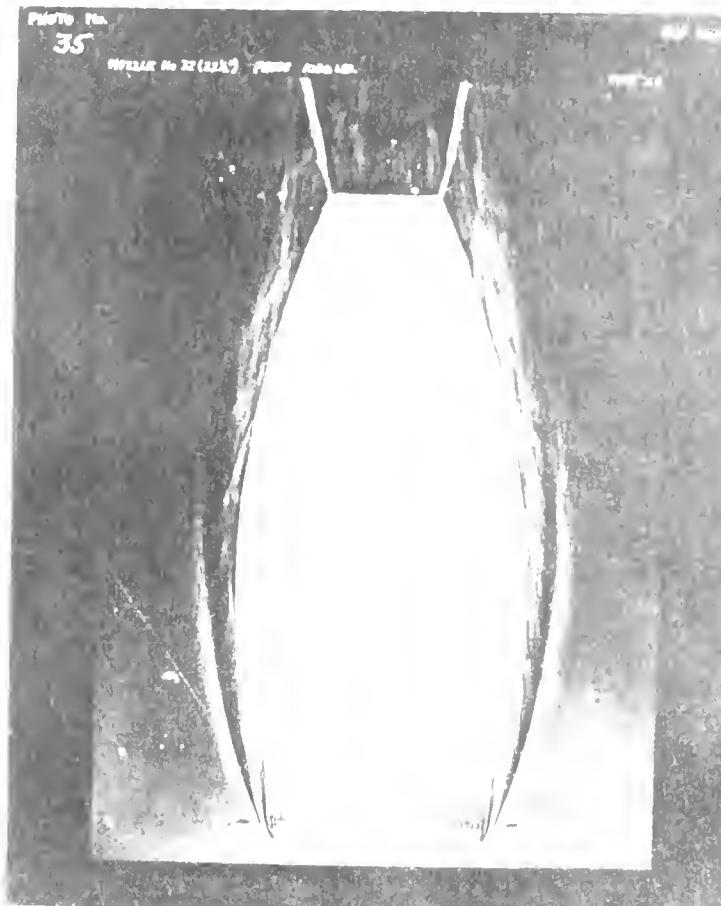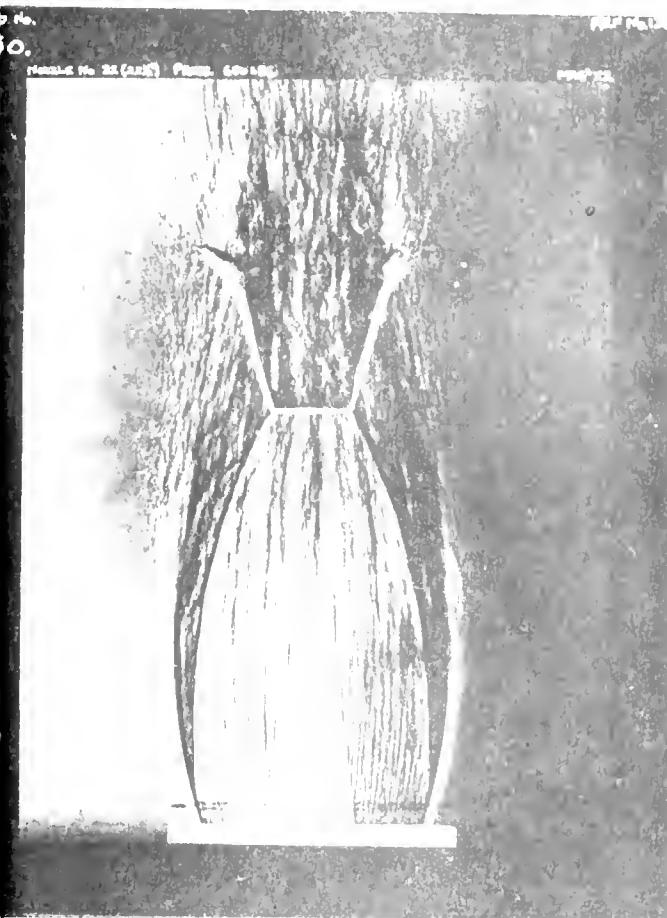
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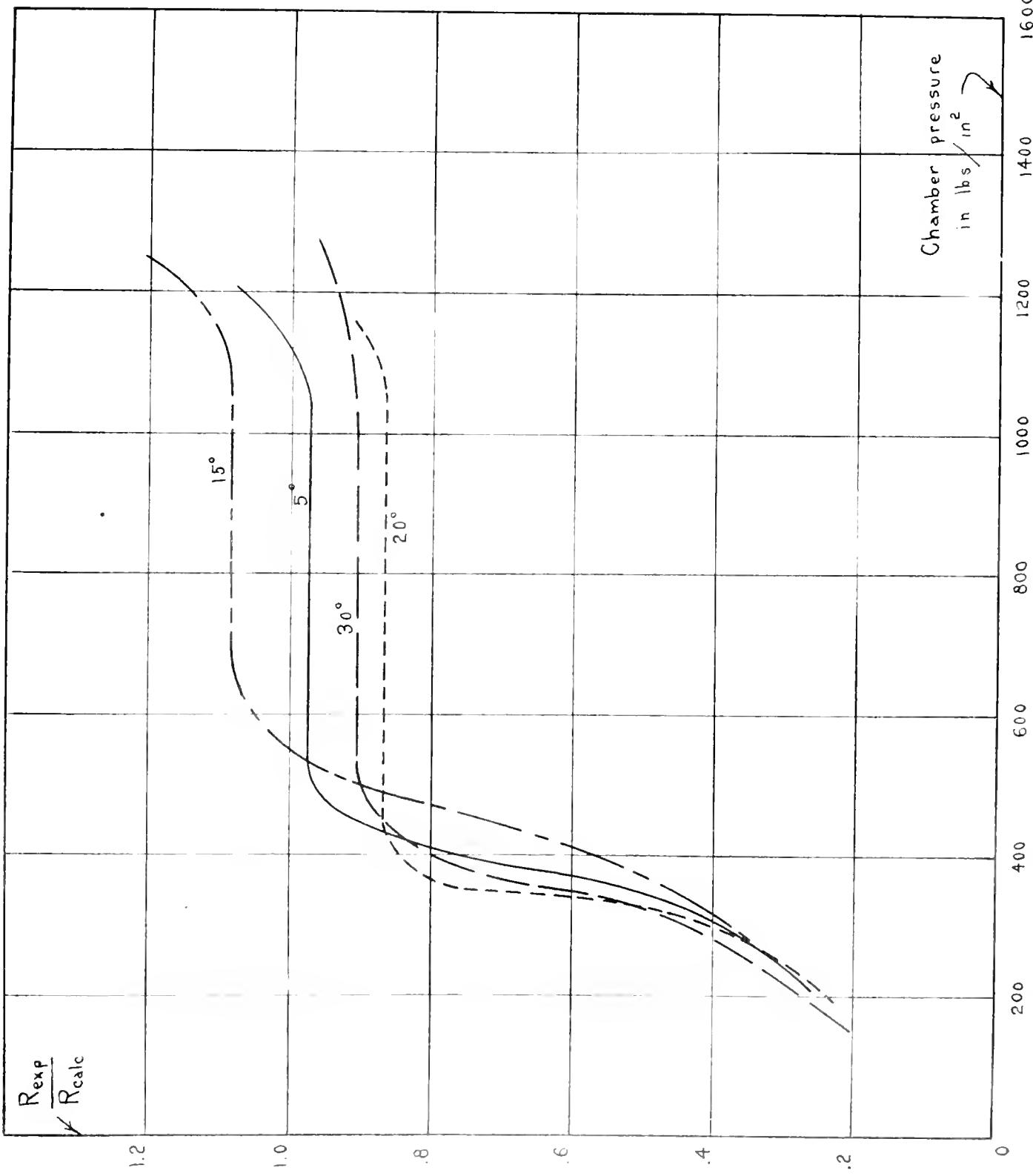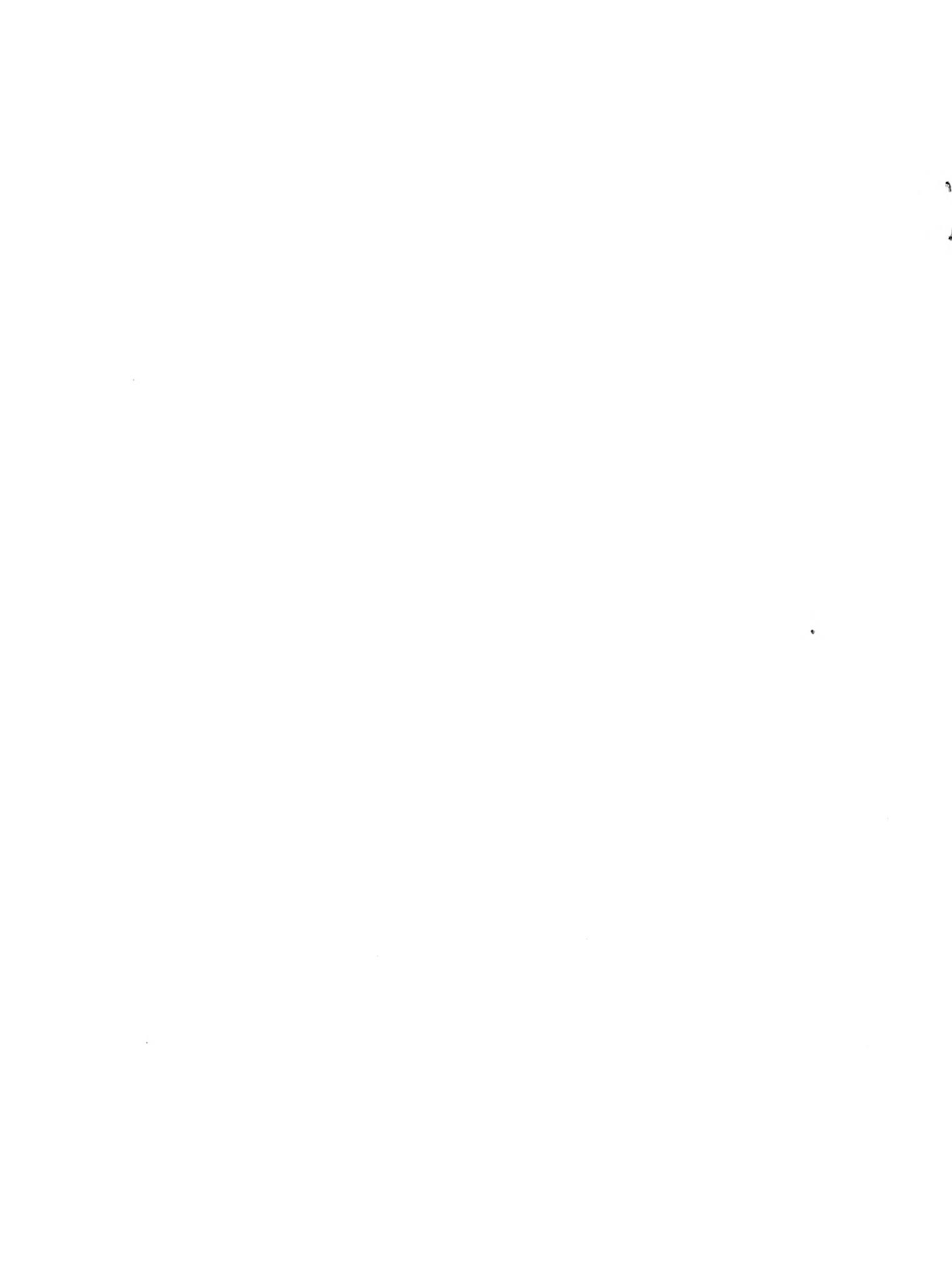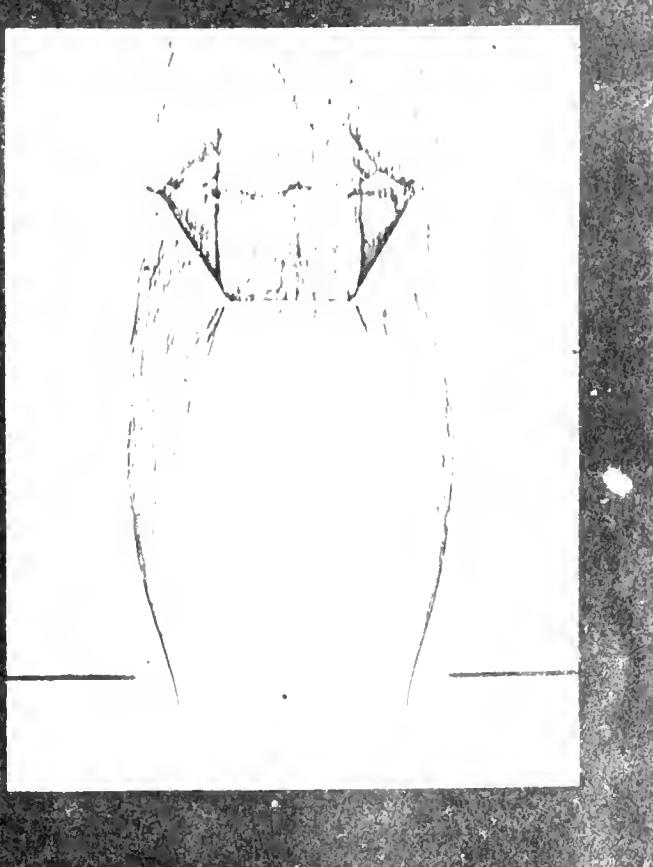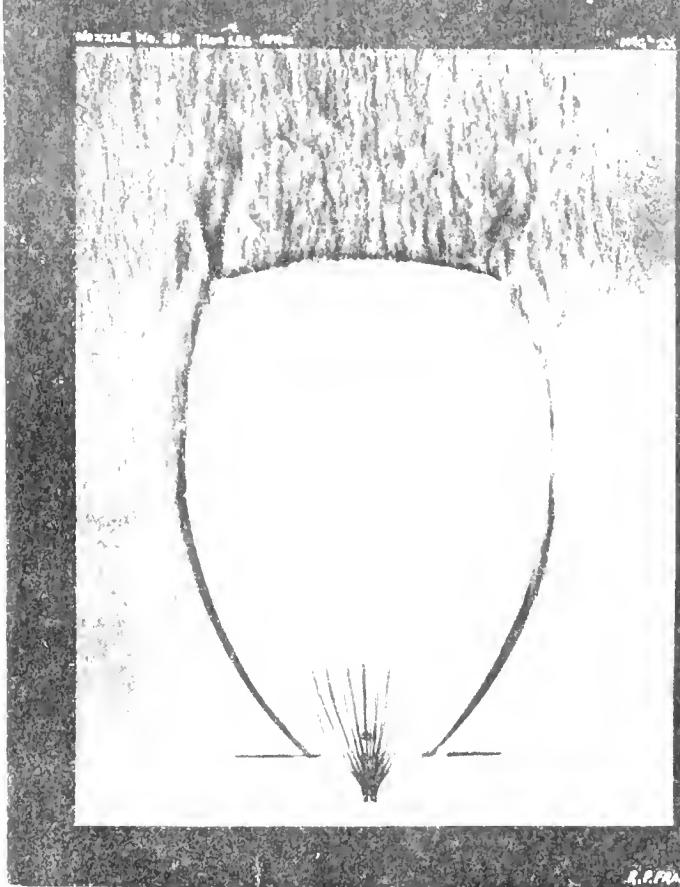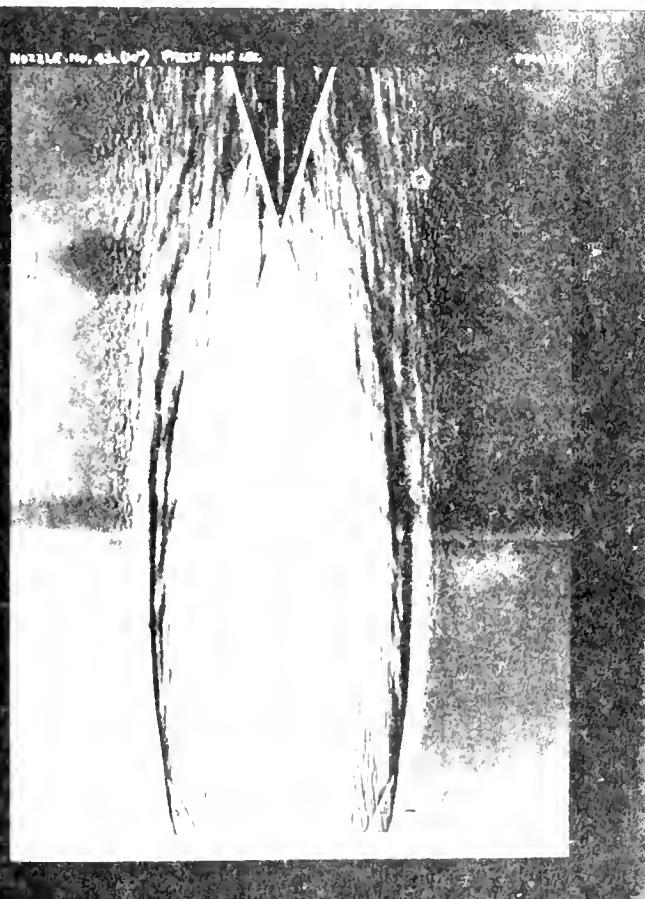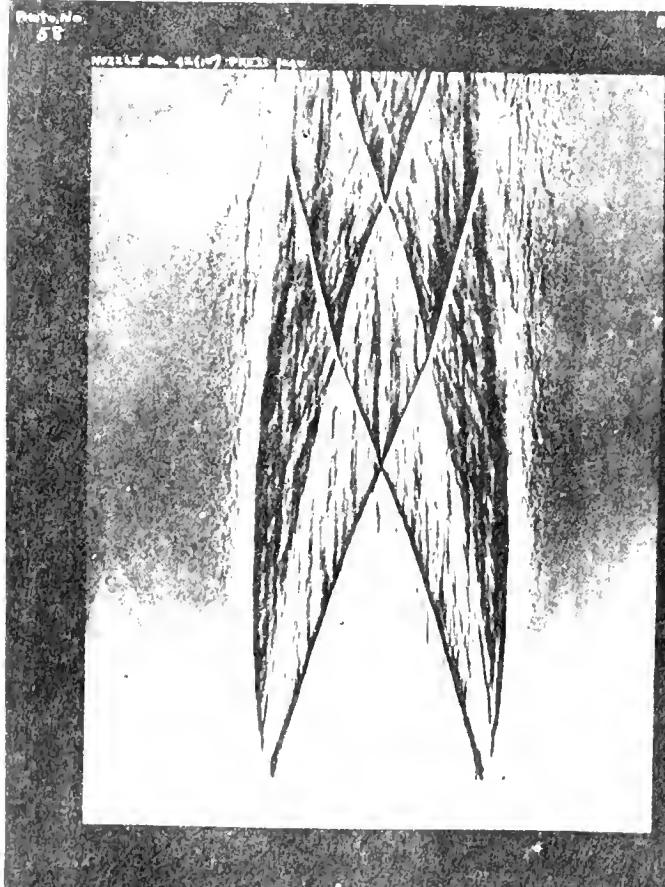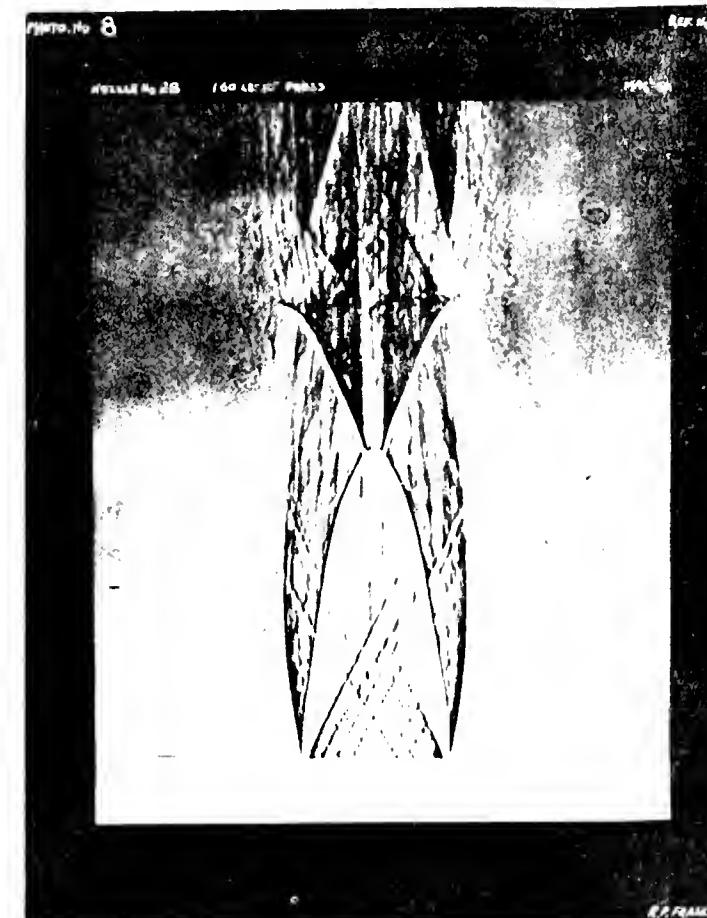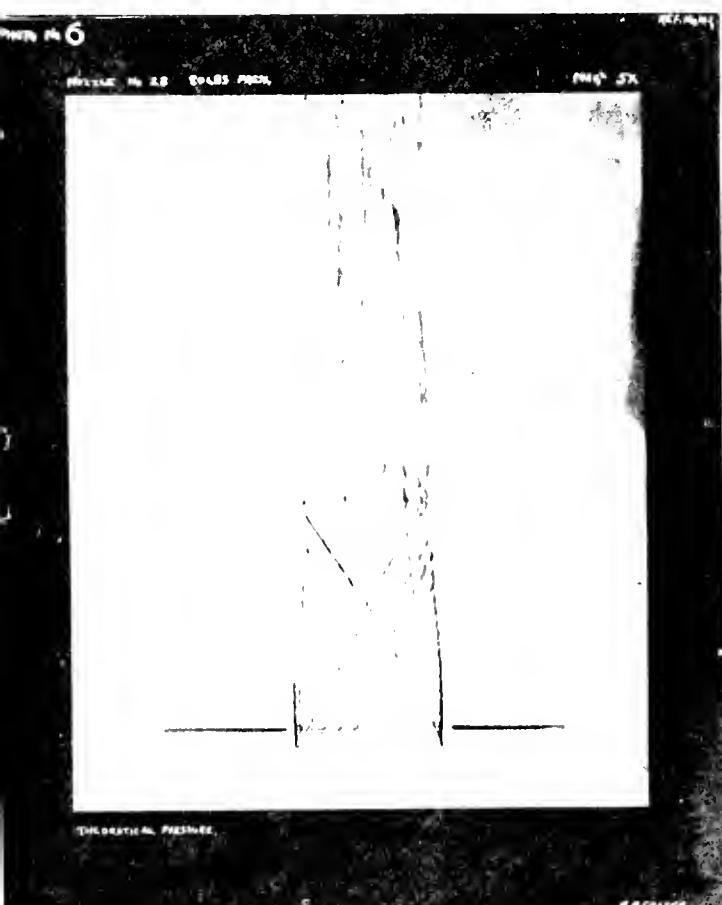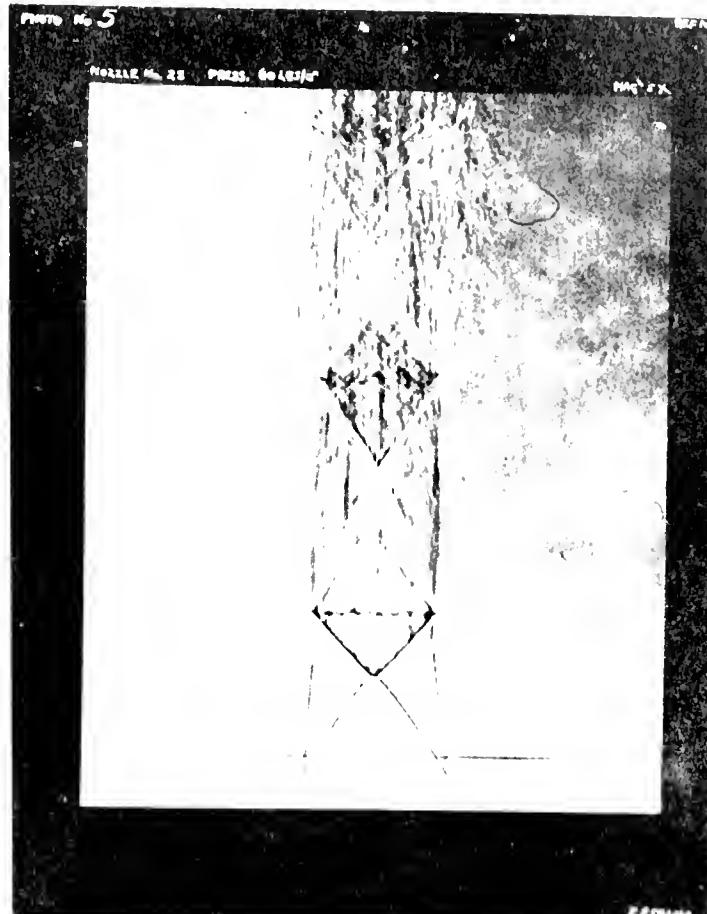
Rexp / Rcalc

Chamber pressure in lbs/in²

15°
5°
30°
20°

TABLE 6

TABLE 7

Date Due

N. Y. U. Institute of
Mathematical Sciences
25 Waverly Place
New York 3, N. Y.